

## Modeling of Infiltration Characteristics by the Modified Kostiakov Method: A Case Study in Thuong River Alluvial Soil in Vietnam

**Nguyen Thi Giang, Ngo Thi Dung & Nguyen Van Dung**

Faculty of Land Management, Vietnam National University of Agriculture, Hanoi 131000, Vietnam

### Abstract

The purpose of this study was to apply the modified Kostiakov method to determine the soil infiltration rate and permeated water amount of Thuong River alluvial soil for orange plantations in Lang Giang district, Bac Giang province, Vietnam. The soil particle size ranged from 0.02 to 2.00mm mainly found in the surface horizon (>77.7%). The soil was slightly acidic in the surface horizon ( $\text{pH}_{\text{KCl}} = 5.11$ ), and highly acidic in the subsurface horizons ( $\text{pH}_{\text{KCl}}$  from 3.42 to 4.79). The organic matter content of the surface horizon was moderate, while it was very low in the other horizons. The total nitrogen (N) content was low (0.15%) in the surface horizon and very low in the subsurface horizons (0.02-0.06%) while the available N was medium. The total phosphorus (P) content in the surface horizon was high (0.4%) and medium in the other horizons. Available P in the surface horizon was high (18.6mg per 100g soil) and decreased in lower depths to only 0.3mg per 100g soil in the deepest (5<sup>th</sup>) horizon. The total and available potassium measurements were very low. A filtration characteristic model was developed by using the modified Kostiakov method for alluvial soil. The constant values  $a$ ,  $\alpha$ , and  $b$  of the equation  $y = at^{\alpha} + b$  were 0.8035, 0.758, and 0.00346, respectively, which were smaller than 1. The average percentage difference between the actual and calculated values by the model was only 0.141%, indicating that the calculated values can accurately predict the actual data measurements in the field. This model would be very helpful for making irrigation schedules and carrying out best water management practices.

**Received:** April 10, 2019  
**Accepted:** August 2, 2019

**Correspondence to**  
nvdung1@vnua.edu.vn

**ORCID**  
Ngo Thi Dung  
<https://orcid.org/0000-0002-0524-0279>  
Nguyen Van Dung  
<https://orcid.org/0000-0002-6057-3455>

### Keywords

Soil permeability, rate of infiltration, actual infiltration, accumulated infiltration

### Introduction

Infiltration is the process of bringing water into the soil profile

the rate of which is dependent on the physical and chemical properties of the water, soil type, soil cover, and porosity, as well as on the state of soil moisture, groundwater table, and time of water infiltration (Johnson, 1963; Michael, 1997; Charbeneau, 2000). The amount of infiltrated water is an important element of the hydrological cycle. As the duration of rainfall increases, the soil permeability reduces (Dagadu & Nimbalkar, 2012) until it becomes saturated, and consequently, generates surface flow (Fetter, 2001; Mahbub *et al.*, 2015). The rate of permeability and amount of infiltration are two important components for calculating the water requirements of a crop that can be applied through an irrigation system. In order to operate the irrigation system effectively, two main issues need to be considered: when and how much water to supply in order to generate water management measurements best suitable for the crops. To date, the problem of measuring infiltration has not been given much attention in Vietnam and elsewhere.

The Kostiakov method is a mathematical formula used to model infiltration. In terms of the importance and economic benefits of water and irrigation management activities, the determination of the constant values  $a$ ,  $\alpha$ , and  $b$  in the mathematical equation to determine the amount of infiltration allows one to assess and predict the amount of irrigation water needed in accordance to the actual water demand of the crop. The objective of this study was to use the modified Kostiakov method in calculating the rates of permeability and water infiltration on the basis of: (1) Determining the coefficients of the Kostiakov method; (2) Evaluating the applicability of the model using field data; and (3) Determining the error between the actual values and the model calculated values. The experiments were conducted in an orange grove in order to determine an appropriate irrigation plan.

## Materials and Methods

### Study site

The study was conducted in Truong Thinh village, Quang Thinh commune, Lang Giang

district, Bac Giang province, Vietnam (21°26'28" N; 106°14'26" E) in 2018. The soil in this area is representative of typical Thuong River alluvial soil. This area was selected for planting Bo Ha oranges since Bac Giang province to restore this specialty orange back to the area.

### Diagram and instrument for soil permeability determination

Currently, there are several methods for measuring soil permeability, but the most common is by using infiltration rings (Fetter, 2001), which is a simple but reliable method due to the improved water supply to keep the water level in the infiltrometer stable during the measurement process.

This method used two infiltration rings, an inner ring with 25cm diameter and outer ring with 50cm diameter. The two rings were concentric and driven into the ground to a depth of 15cm (**Figure 1a**). The outer ring was used as a buffer pond to avoid the lateral movement of water and to maintain the water within the inner ring with vertical permeability only. The water supply for the infiltration process was a tube filled with water with two regulating valves, which were suspended on a scale to determine the water volume in the pipe.

Three infiltrometers (infiltration rings) were installed lengthwise with a distance of 40cm between them (**Figure 1b**).

### Measurements and calculations

In order to measure the rates of permeability and water infiltration, the water resource pipe was first filled with water. The pipe had two valves. Valve 1 supplied water to the pipe while valve 2 supplied water to the infiltration rings. For the infiltration measurement, valve 1 was closed while valve 2 was opened. The water inside the pipe only flowed down to the infiltration rings when the valve 2 opening was in contact with air. When the water reached a constant level, the opening of valve 2 was not in contact with the air and water stopped flowing. The amount of water was measured at time 1 (*a*) and after 5min at time 2 (*b*). The permeated water

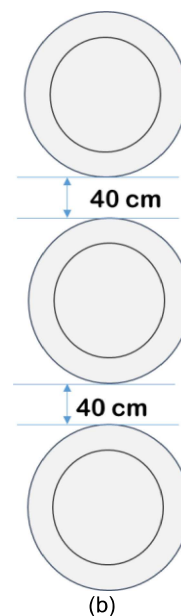
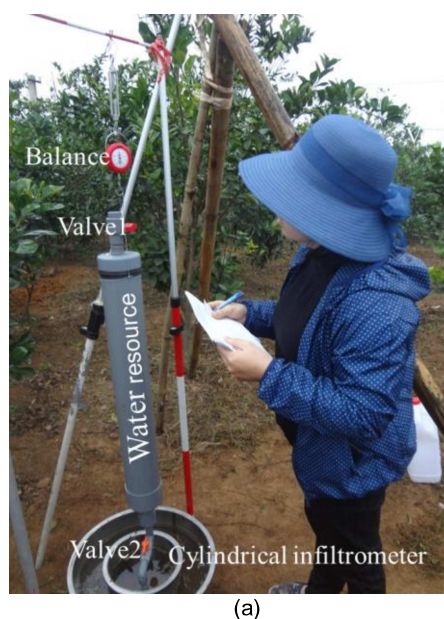


Figure 1. Installation of infiltrmeters in the field and its components

was calculated as the difference in the amount of water between time 1 and time 2 ( $a-b$ ).

The amount of infiltrated water was also calculated. Given that the density of water is  $1 \text{ g cm}^{-3}$ , the volume of infiltrated water was calculated as the weight (amount of water) divided by the specific gravity according to the following equation (1):

$$V = \frac{m}{\varrho} \quad (1)$$

where  $V$  is the water volume ( $\text{cm}^3$ ),  $m$  is the infiltration water weight ( $m = a - b \text{ (g)}$ ), and  $\varrho$  is the specific gravity equal to  $1 \text{ g cm}^{-3}$ . The depth of the permeable water layer was calculated as the volume of water divided by the area of the infiltration ring according to the following equation (2):

$$h = \frac{V}{A} \quad (2)$$

where  $h$  is the permeable water depth (cm),  $V$  is the water volume ( $\text{cm}^3$ ), and  $A$  is the cross section area of infiltration ring ( $\text{cm}^2$ ).

### Determining the coefficient in the permeability equation

The relationship of the accumulated infiltration  $y$  with respect to time  $t$  was

mathematically determined using the following equation (3) from the modified Kostiakov method:

$$Y = at^\alpha + b \quad (3)$$

where  $Y$  is the accumulated infiltration at time  $t$  (cm) and  $t$  is the infiltration time (min).

The coefficients  $a$ ,  $\alpha$ , and  $b$  were calculated following the method of Davis (1943). The values of  $a$ ,  $\alpha$ , and  $b$  are usually less than 1 (Michael, 1997). Equation (3) was calculated in three steps by: (i) determining  $Y$  and  $t$  according to real measurements; (ii) selecting the pairs of values for the time and amount of infiltrated water after 5 minutes ( $t_1 y_1$ ) and for the time and amount of infiltrated water when the soil was finally saturated with water ( $t_2 y_2$ ); and (iii) calculating a third value for  $t_3$  using the values of  $t_1$  and  $t_2$  via equation (4):

$$t_3 = \sqrt{t_1 t_2} \quad (4)$$

The value of  $t_3$  was used to determine  $Y_3$ , and then applied to following the equation (5) to calculate  $b$ :

$$Y_1 = at_1^\alpha + b; Y_2 = at_2^\alpha + b$$

$$Y_1 Y_2 = (at_1^\alpha + b) \times (at_2^\alpha + b)$$

$$Y_1 Y_2 = a(\sqrt{t_1 t_2})^{\alpha^2} + ab(t_1^\alpha + t_2^\alpha) + b^2$$

$$Y_1 Y_2 = a(\sqrt{t_1 t_2})^{\alpha^2} + 2ab\sqrt{t_1 t_2}^{\alpha} + b^2$$

$$Y_1 Y_2 = a(\sqrt{t_1 t_2})^{\alpha^2} + 2a\left[\sqrt{t_1 t_2}\right]^{\alpha} b + b^2$$

$$Y_1 Y_2 = a\left[\sqrt{t_1 t_2}\right]^{\alpha} + b^2;$$

$$\text{Set } Y_3^2 \leq Y_1 Y_2, \text{ in which } Y_3 = \sqrt{Y_1 Y_2} \text{ (5).}$$

### Norms and methods of soil analysis

The soil analysis was conducted at the Central Laboratory, Faculty of Land Management, Vietnam National University of Agriculture. The properties of the soil were measured as follows: pH<sub>KCl</sub> was measured with a pH meter; OC% was measured using the Walkley and Black method; total N was measured by the Kjeldhal method; total P<sub>2</sub>O<sub>5</sub> was measured by the colorimetry method; total K<sub>2</sub>O was measured by the flame photometer method after digesting the soil in HClO<sub>4</sub>, HF, and HCl acids and then measuring the K content; hydrolysis N was measured with the Tiurin and Kononova method; available P<sub>2</sub>O<sub>5</sub> was measured by the Oniani method; available K<sub>2</sub>O was measured by the Matslova method via a flame photometer; soil texture was measured with pipette methods; particle density was measured by the picnometer method; and bulk density was measured by the core method (Van Reeuwijk, 1986).

### Soil moisture measurements

In this research, soil moisture was determined by the weighing method in which the soil samples were dried at 105°C for 48h, weighed, redried, and weighed again until the mass of the samples were constant. The mass of water lost was calculated as a percentage of the dried soil.

## Results and Discussion

### Soil moisture

It was found that the soil moisture content before and after the experiment was 27.07% and 37.02%, respectively.

### Soil properties

Soil profiling showed five distinct layers (**Figure 2**). Generally, the soil had a particle size component ranging from 0.02 to 2.00mm, present mostly in surface layer (77.7%), then decreased with depth and the smallest amount was in the 4<sup>th</sup> and 5<sup>th</sup> layers. Soil particle density did not change in layers. Organic matter content was the highest in surface layer and decreased with depth, was lowest in the 5<sup>th</sup> layer. Soil bulk density did not change much among layers (**Table 1**).

The soil was slightly acidic in the first layer to very acidic in the sublayers (**Table 1**). The organic matter content of the surface layer was medium, while it was very low for the rest of the layers. The total N was low in the topsoil and much lower in the sublayers. The available N was medium. The total P was high in the soil surface and medium in the other layers. Available P was high in the surface layer but gradually decreased in lower depths. The total K, as well as available K, were low regardless of the layer.



**Figure 2.** Soil profile at the infiltration measurement site

## Calculated coefficients in the Kostiakov equation

### Infiltrations

The average observed infiltration rate decreased rapidly over the first 30min and steadily decreased from 30-110min before evening out for the remaining experimental time. The amount of stable seepage water after 135min reached 5.79 cm h<sup>-1</sup> (Figure 3).

### Calculated coefficients in the Kostiakov equation

The calculated value for  $t_3$  was determined using the values of  $t_1$  and  $t_2$ , with  $t_1 = 5$  and  $t_2 = 135$ min. The value of  $t_3$  was 25.98min using the equation:

$$t_3 = \sqrt{t_1 \times t_2} = \sqrt{5.0 \times 135} = 25.98 \text{ min}$$

The average value of  $y_3$  was calculated from equation (5) with the measurement from the first 5min ( $y_1 = 2.44$ ) and at saturation ( $y_2 = 30.52$ ) (Table 2) using the equation:

$$y_3 = \sqrt{y_1 \times y_2} = \sqrt{2.44 \times 30.52} = 8.63 \text{ cm}$$

The calculated value of  $b$  was **0.00346** based on equation (3):

$$b = \frac{y_1 y_2 - y_3^2}{y_1 + y_2 - 2y_3} = \frac{2.44 \times 30.52 - 8.63^2}{2.44 + 30.52 - 2 \times 8.63} = 0.00346$$

### The calculated values of $a$ and $\alpha$

Measurements were taken every 5min, and thus, there were 27 measurements recorded at the end when the soil was saturated with water. Ten

measurements were selected randomly from Table 2 and used for the exclusion  $a$  to find  $\alpha$ . After deriving  $\alpha$ , the value was used in the equation to further calculate the value  $a$ . Equation (3) was rearranged and written as:  $Y - b = at^\alpha$ , and calculating the log of both sides resulted in equation (6):

$$\text{Log}(Y - b) = \log(a) + \alpha \log(t) \quad (6)$$

The values of  $y$  and  $t$  were determined from Table 2 and the value of  $b$  was entered into equation (6) to get the corresponding equations from (7) to (16):

$$\text{Log}(2.44 - 0.00346) = 0.39 = \log a + \alpha \log(5) \quad \text{or} \quad \log a + 0.699 \alpha \quad (7)$$

$$\text{Log}(4.51 - 0.00346) = 0.65 = \log a + \alpha \log(10) \quad \text{or} \quad \log a + 1.000 \alpha \quad (8)$$

$$\text{Log}(6.37 - 0.00346) = 0.80 = \log a + \alpha \log(55) \quad \text{or} \quad \log a + 1.176 \alpha \quad (9)$$

$$\text{Log}(9.68 - 0.00346) = 0.99 = \log a + \alpha \log(25) \quad \text{or} \quad \log a + 1.398 \alpha \quad (10)$$

$$\text{Log}(15.45 - 0.00346) = 1.19 = \log a + \alpha \log(45) \quad \text{or} \quad \log a + 1.653 \alpha \quad (11)$$

$$\text{Log}(20.40 - 0.00346) = 1.31 = \log a + \alpha \log(65) \quad \text{or} \quad \log a + 1.813 \alpha \quad (12)$$

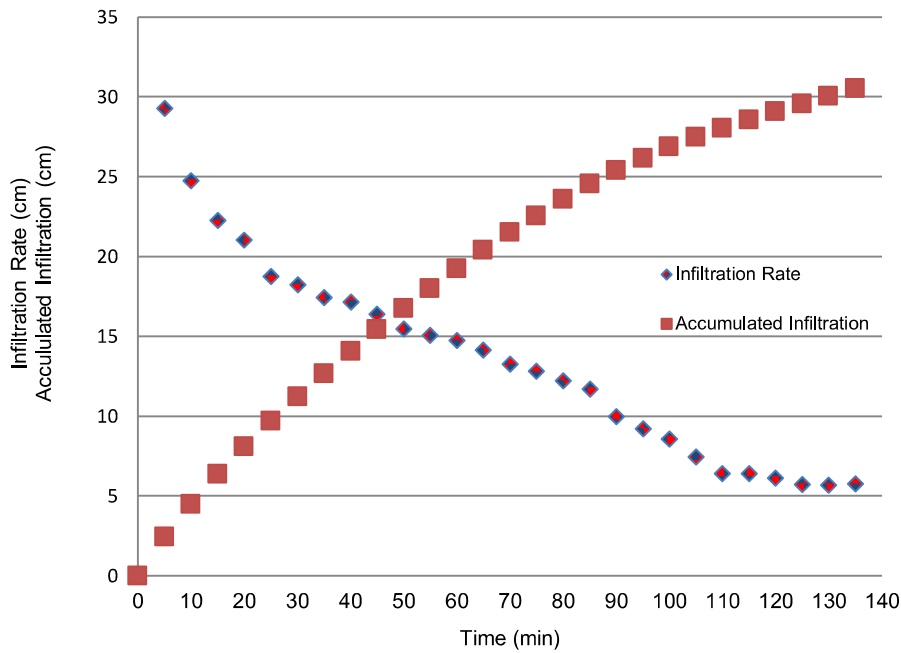
$$\text{Log}(24.57 - 0.00346) = 1.39 = \log a + \alpha \log(85) \quad \text{or} \quad \log a + 1.929 \alpha \quad (13)$$

$$\text{Log}(27.51 - 0.00346) = 1.44 = \log a + \alpha \log(105) \quad \text{or} \quad \log a + 2.021 \alpha \quad (14)$$

$$\text{Log}(29.09 - 0.00346) = 1.46 = \log a + \alpha \log(120) \quad \text{or} \quad \log a + 2.079 \alpha \quad (15)$$

Table 1. Soil physical-chemical properties

Soil layers	pH <sub>KCl</sub>	OC	N	P <sub>2</sub> O <sub>5</sub>	K <sub>2</sub> O	N	P <sub>2</sub> O <sub>5</sub>	K <sub>2</sub> O	Texture (%)			Particle density	Bulk density (g cm <sup>-3</sup> )
		%				mg/100g			<0.002mm	0.002-0.02mm	0.02-2.0mm		
T1	5.11	1.44	0.15	0.40	0.35	6.0	18.6	7.7	9.5	12.8	77.7	2.59	1.20
T2	3.42	0.42	0.06	0.08	0.38	5.6	7.4	25.8	17.1	15.1	67.8	2.65	1.21
T3	3.43	0.38	0.04	0.06	0.42	4.2	3.9	25.8	21.5	17.3	61.2	2.62	1.21
T4	3.74	0.11	0.03	0.06	0.68	5.3	1.0	11.1	36.5	13.5	50.0	2.58	1.29
T5	3.79	0.07	0.02	0.05	0.76	8.1	0.3	7.7	34.3	14.9	50.8	2.58	1.29



**Figure 3.** Mean infiltration rate (cm h<sup>-1</sup>) and accumulated infiltration (cm) throughout the experiment time (min)

**Table 2.** Average values of accumulated infiltration, infiltration rate vs. time for infiltrometers 1, 2, and 3

Time ( <i>t</i> , min)	Rate of infiltration (cm h <sup>-1</sup> )	Accumulated infiltration ( <i>y</i> ; cm)
0	0.00	0.00
5	29.30	2.44
10	24.78	4.51
15	22.30	6.37
25	18.78	9.68
45	16.38	15.45
65	14.15	20.40
85	11.70	24.57
105	7.47	27.51
120	6.12	29.09
135	5.79	30.52

$$\text{Log } (30.52 - 0.00346) = 1.48 = \log a + \alpha \log (135) \text{ or } \log a + 2.130 \alpha \quad (16)$$

The sum of the first five equations (equations 7 to 11) was used to get equation (17):

$$5 \log a + 5.926 \alpha = 4.019 \quad (17)$$

The sum of the five subsequent equations (equations 12 to 16) was used to get equation (18):

$$5 \log a + 9.973 \alpha = 7.088 \quad (18)$$

To calculate the values of equations (17) and (18),  $\alpha$  was substituted with **0.758** while the value of  $\log(a)$  was -0.095 and  $a$  was **0.8035**. Thereafter,  $a$ ,  $b$ , and  $\alpha$  were replaced with these values in the equation for individual elapsed times:  $\log(y - b) = \log(a) + \alpha \log(t)$  to produce ten equations (equations 19 to 28), determine the value of  $y$  (from  $y_{5 \text{ min}}$  to  $y_{135 \text{ min}}$ ):

$$\text{At } t = 5 \text{ min, } \log(y_{5 \text{ min}} - 0.00346) = -0.095 + 0.5301 = 0.4350; \quad y_{5 \text{ min}} = 2.7229 \quad (19)$$

$$\text{At } t = 10\text{min}, \log(y_{10\text{min}} - 0.00346) = -0.095 + 0.7583 = 0.6633; \quad y_{10\text{min}} = 4.6059 \quad (20)$$

$$\text{At } t = 15\text{min}, \log(y_{15\text{min}} - 0.00346) = -0.095 + 0.8919 = 0.7968; \quad y_{15\text{min}} = 6.2640 \quad (21)$$

$$\text{At } t = 25\text{min}, \log(y_{25\text{min}} - 0.00346) = -0.095 + 1.2537 = 0.9651; \quad y_{25\text{min}} = 9.2275 \quad (22)$$

$$\text{At } t = 45\text{min}, \log(y_{45\text{min}} - 0.00346) = -0.095 + 1.3748 = 1.1587; \quad y_{45\text{min}} = 14.4101 \quad (23)$$

$$\text{At } t = 65\text{min}, \log(y_{65\text{min}} - 0.00346) = -0.095 + 1.4631 = 1.2798; \quad y_{65\text{min}} = 19.0447 \quad (24)$$

$$\text{At } t = 85\text{min}, \log(y_{105\text{min}} - 0.00346) = -0.095 + 1.5327 = 1.3681; \quad y_{105\text{min}} = 23.3413 \quad (25)$$

$$\text{At } t = 105\text{min}, \log(y_{105\text{min}} - 0.00346) = -0.095 + 1.5327 = 1.4377; \quad y_{105\text{min}} = 27.3979 \quad (26)$$

$$\text{At } t = 120\text{min}, \log(y_{120\text{min}} - 0.00346) = -0.095 + 1.5767 = 1.4817; \quad y_{120\text{min}} = 30.3176 \quad (27)$$

$$\text{At } t = 135\text{min}, \log(y_{135\text{min}} - 0.00346) = -0.095 + 1.6155 = 1.5205; \quad y_{135\text{min}} = 33.1502 \quad (28)$$

#### Calculated percentage of error

Using equation (29) to calculate the error between the actual measured values and the values calculated by Kostiakov method, the error values were calculated according to each time interval measured from equations (19) to (28). The calculated values are presented in **Table 3**.

$$\text{Error} = \sum_{i=1}^n \left| \frac{AI - CI}{AI} \right| \times 100 \quad (29)$$

where *AI* is the actually accumulated

infiltration, *CI* is the calculated accumulated infiltration by the model, and *i* is the number of data points.

**Table 3** shows the calculated values against actual values within the allowed range and the percentage of error between the actual and the calculated values of the accumulated infiltration time. Error and coefficient *a*; *a* and *b* were both smaller than 1. This result was in accordance with previous research (Michael, 1997; Mahbub *et al.*, 2015).

**Figure 4** shows that the values calculated from the Kostiakov method can accurately predict the actual field data measurements. Therefore, this model provides an accurate and useful basis for determining the infiltration rate and calculating the amount of infiltrated water in the field. Simultaneously, the research results can provide a basis for irrigation scheduling for plants to achieve high water use efficiency when making decisions about irrigation time, total water amount for each irrigation, and the number of irrigations in a cultivation period.

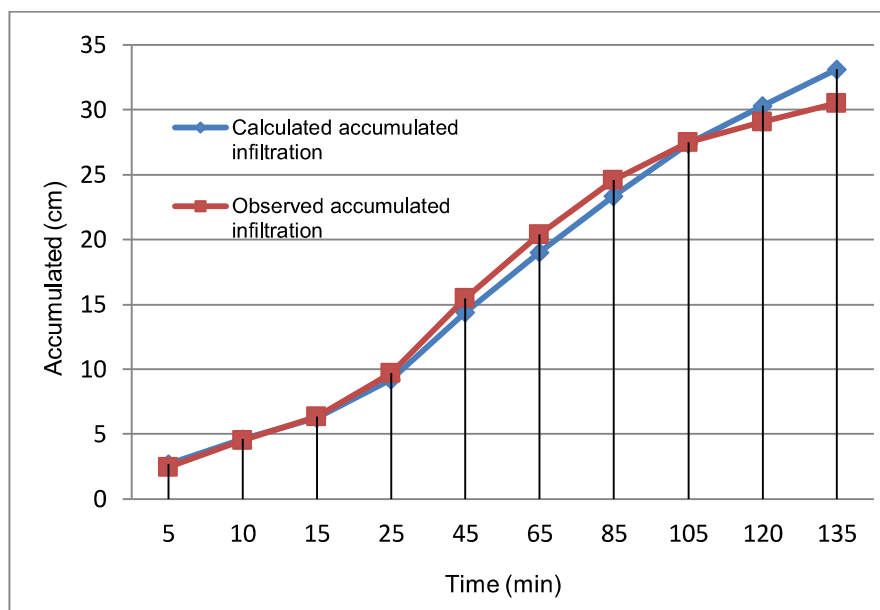
## Conclusions

Alluvial soil of the Thuong River had particle sizes of 0.02-2.00mm mainly present in the surface layer (>77.7%). The soil was slightly acidic ( $\text{pH}_{\text{KCl}} = 5.11$ ) and became more acidic in lower depths. The organic matter content of the

**Table 3.** The calculated percentage of error between the actual and calculated values of accumulated infiltration vs. time

Time (min)	Observed calculated infiltration (cm)	Calculated accumulated infiltration (cm)	Percent of error (%)
5	2.442	2.7229	11.517
10	4.507	4.6059	2.195
15	6.365	6.2640	-1.590
25	9.684	9.2275	-4.711
45	15.451	14.4101	-6.737
65	20.404	19.0447	-6.660
85	24.574	23.3413	-5.015
105	27.511	27.3979	-0.410
120	29.091	30.3176	4.216
135	30.525	33.1502	8.602
Average			0.141





**Figure 4.** The observed and calculated accumulated infiltration through time (min)

surface layer was average, while the remaining layers were very low. The total N was poor in the topsoil and gradually decreased to very poor levels in the sub-layers while the available N was medium. The total P was high in the surface (0.4%) and medium in the subsurface, while the available P was high in the surface layer but gradually reduced in lower depths. The total available K was poor.

The values of  $a$ ,  $\alpha$ , and  $b$  used in the modified Kostiakov equation were 0.8035, 0.758 and 0.00346, respectively, which were all less than 1. The percentage error between the calculated values and the actual measured data was low (0.141%) indicating that the mathematical values were within the acceptable range that can accurately predict the actual field data measurements. Thus, the Kostiakov method was useful in calculating the infiltration rate and the amount of infiltrated water into the soil to provide useful information in water management for plants.

## References

Charbeneau R. J. (2000). Groundwater Hydraulics and

Pollutant Transport. Prentice Hall, Upper Saddle River, New Jersey.

Davis D. S. (1943). Empirical Equations and Monography. Hill Book Co., New York, 200.

Dagadu J. S. & Nimbalkar P. T. (2012). Infiltration studies of different soil under different soil conditions and comparison of infiltration models with field data. IJAET, III, 154-157.

Fetter C. W. (2001). Applied Hydrogeology (4<sup>th</sup> ed.). Prentice Hall, Upper Saddle River, New Jersey.

Johnson A. I. (1963). A field method for measurement of infiltration. Geological survey water-supply paper. United States Government Printing Offices, Washington D. C.

Mahbub H., Tamara C., Mebougna D. & Aschales K. (2015). Modeling of infiltration characteristics by modified Kostiakov method. Journal of Water Resource and Protection. Scientific Research. 7: 1309-1317. Retrieved from <http://dx.doi.org/10.4236/jwarp.2015.716106> on September 11, 2017.

Michael A. M. (1997). Irrigation: Theory and Practice. Vikas Publishing House PVT Ltd., Delhi.

Van Reeuwijk L. P. (1986). Procedures for Soil Analysis. National Soil Reference and Information Centre-ISRIC, Food and Agriculture Organization of the United Nations (FAO).