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Application of Mathematical Models to Predict Growth and Optimum Slaughter Ages of Na Tau Ducks in Vietnam

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Abstract

This study was aimed to analyze the growth curves and predict the optimum slaughter ages of Na Tau ducks. Six mathematical models, namely von Bertalanffy, Janoschek, Gompertz, Logistic, Lopez, and Richards, were employed. Body weight data were collected weekly from birth to 10 weeks of age. The parameters used to evaluate the growth functions were the coefficient of determination (R^2), the AIC (Akaike's information criterion), BIC (Bayesian information criterion), and the standard error of the regression function (SER), as analyzed using R software. Among the models tested, the Janoschek model provided the best fit for both male and female ducks, as it exhibited the lowest AIC, BIC, and SER values, together with the highest R^2. The asymptotic body weights of male ducks ranged from 2213.16g (Logistic model) to 2726.58g (Lopez model), whereas for female ducks, they were estimated between 1968.89g (Logistic model) and 2401.25g (Lopez model). The maximum average weekly gains (AWG_{max}) estimated by the best-fit model were recorded at 6.28 and 5.81 weeks for males and females, respectively. Based on these findings, the optimal slaughter ages were determined to be 6.28 weeks for male Na Tau ducks (with a body weight of 1617.34g) and 5.81 weeks for females (with a body weight of 1410.9g).

Keywords

Na Tau ducks, mathematical models, best-fit, growth curve, slaughter age

Introduction

Duck farming plays a very important role in the socio-economic life of Vietnam. The number of ducks in the country in was around 87 million, ranking Vietnam as second in the world. As for duck meat

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output, Vietnam also ranks in the top 10 countries with the largest output in the world (FAO, 2025). Raising ducks is closely associated with wet rice culture and has been a livelihood for Vietnamese farmers for many generations. Today, despite the great influence of the rural urbanization process that has reduced the area of agricultural land and pastures, industrial duck farming has an increasing trend and makes important contributions to job creation, social security, and the sustainable economic development of the country. In duck farming, Vietnamese native duck farming is important in the preservation of genetic resources to provide starting materials for scientific research and breed improvement, ensuring the maintenance of biodiversity.

Body weight is one of the most important traits of livestock because it is strongly associated with other economically valuable characteristics of production and reproduction (Salem *et al.*, 2013; Amraei *et al.*, 2017). Today, modelling growth curves is useful to predict the weight and weight gain of an animal throughout its life, thereby supporting feeding and management as well as estimating the suitable slaughter age. Studies have revealed that a delay in the optimal slaughter weight is negatively associated with the profitability of the system (Wolfová *et al.*, 2009). Therefore, the ability to estimate the slaughter weight of an animal for a given age would allow farmers to adopt strategies that would improve their profitability (Ozturk *et al.*, 2023).

Several studies estimating the biological parameters of the growth curves for ducks have been reported. Arai *et al.* (1983) used two non-linear regression functions to describe the growth patterns of male and female spot-billed ducks over time and concluded that the Gompertz curves better explained the data than the Logistic ones. Knítová *et al.* (1991) described the growth curves of nine selected duck lines and one random control population breed and found that at lower weight gains, the estimations of the parameters of the Richards function were relatively reliable. Gille & Salomon (1994) showed the Janoschek model yielded the least residual sums of squares for humerus mass and the Richards model for femur mass with only minor differences between both equations when compared with the von Bertalanffy, Gompertz, and Logistic growth curves using bone weight data in White Pekin ducks. In a study with four selected duck lines, Maruyama *et al.* (1999) found the Weibull function yielded the best goodness of fit, among sigmoidal functions, for the body and carcass weights of male ducks selected for market weight. Asiamah *et al.* (2020) reported the von Bertalanffy was the best fitting model for the overall growth of Leizhou Black ducks. Thinh *et al.* (2021) indicated that the Gompertz was the best growth model for modelling the growth of Eastern spot-billed ducks in Vietnam.

Na Tau ducks are an indigenous breed originating from the northwestern mountainous region of Vietnam, particularly from Dien Bien province. They are well-adapted to the local climate, low-input traditional farming systems, and extensive grazing practices common in upland rural communities. However, they remain under-researched compared to other indigenous or commercial breeds. In addition, there has been no prior research employing mathematical models to estimate the optimal slaughter age for indigenous duck breeds in Vietnam. Therefore, this study aimed to evaluate the predictive performance of six mathematical models (von Bertalanffy, Janoschek, Gompertz, Logistic, Lopez, and Richards) in forecasting growth and identifying the most suitable slaughter age for Na Tau ducks in Vietnam.

Materials and Methods

Animals and experimental site

The animals used for this study were Na Tau ducks, an indigenous genetic resource native to Vietnam. These ducks originated from Na Tau commune, located in Dien Bien district, Dien Bien province, in the northwestern region of Vietnam (21°23′N, 103°01′E). For the purposes of this research, all individuals were reared under controlled conditions at the Experimental Animal Farm of the Faculty of Animal Science, Vietnam National University of Agriculture (21°01′08″N, 105°56′14″E).

Animal management, data collection, and statistical analysis

The ducks were reared in open-sided cages with rice husks used as the bedding material. During the first two weeks of life, the birds were exposed to continuous lighting (24 hours/day), after which they were maintained under natural light conditions. All the ducks were raised under the same care and feeding conditions. Each individual was permanently identified using a leg band and was weighed weekly throughout the experimental period. Commercial feed and water were provided *ad libitum*. The dietary composition was as follows: from days 1 to 21, the feed contained 2900 kcal ME kg^{-1}, 20% crude protein (CP), and 4% crude fiber (CF); and from days 22 to 70, the feed contained 2800 kcal ME/kg, 18% CP, and 7% CF.

To record the body weights, a total of 120 ducks were weighed weekly from birth to 10 weeks of age, prior to morning feeding. The growth dataset was processed using R software (version 4.2.2). A variety of nonlinear mathematical models were employed to fit the growth curves, namely von Bertalanffy (von Bertalanffy, 1957), Janoschek, Gompertz (Gompertz, 1825), Logistic (Pearl, 1977), Lopez (Lopez *et al.*, 2000), and Richards (Richards & Kavanagh, 1945) (**Table 1**).

The parameters used in the growth model were BW_0, which represents the initial body weight of the animal (g); and α, the maximum or upper asymptotic body weight (g), which is the weight the animal will eventually achieve once fully grown. The parameter k defines the growth rate reductions as the animal nears its maximum weight. The parameter β represents the growth phase before the inflection point on the curve, while m is the shape parameter determining the position of the curve point. These parameters were estimated using the nlsLM() function within minpack.lm package (Elzhov *et al.*, 2016) of R 4.2.2 software.

To determine the most appropriate model for fitting the duck growth curves, several statistical criteria were applied, namely the Akaike information criterion (AIC), Bayesian information criterion (BIC), standard error of the regression function (SER), and the coefficient of determination (R^2). All calculations were performed using R software. The AIC, BIC, and SER values were ranked from lowest (rank = 1) to highest (rank = 6), while the R^2 values were ranked from highest (rank = 1) to lowest (rank = 6). The optimal model was identified as the one with the lowest AIC, BIC, and SER values, and the highest R^2 (Ramos *et al.*, 2013; Bo *et al.*, 2023).

Table 1. Growth curve models used for estimating body weight, weekly gain (WGt) and average weekly gain (AWGt)

No.	Model	Function	The first derivative of functions (WGt)	Average weekly gain (AWGt)
1	von Bertalanffy	$BW_t = \alpha \times (1 - \beta \times e^{-kt^3})$	$3\alpha k\beta t^2 e^{-kt^3}$	$\frac{\alpha\beta(1-e^{-kt^3})}{t}$
2	Janoschek	$BW_t = \alpha - (\alpha - BW_0) \times e^{-kt^m}$	$(\alpha - BW_0)kmt^{(m-1)}e^{-kt^m}$	$\frac{(\alpha - BW_0)(1 - e^{-kt^m})}{t}$
3	Gompertz	$BW_t = \alpha \times e^{-\beta \times e^{-kt}}$	$\alpha k\beta e^{-kt}e^{-\beta e^{-kt}}$	$\frac{\alpha(e^{-\beta e^{-kt}} - e^{-\beta})}{t}$
4	Logistic	$BW_t = \frac{\alpha}{1 + \frac{\alpha - BW_0}{BW_0} \times e^{-kt}}$	$\frac{\alpha(\frac{\alpha - BW_0}{BW_0})ke^{-kt}}{(1 + \frac{(\alpha - BW_0)}{BW_0}e^{-kt})^2}$	$\frac{(\alpha - BW_0)(1 - e^{-kt})}{t(1 + \frac{(\alpha - BW_0)}{BW_0}e^{-kt}}$
5	Lopez	$BW_t = \frac{(BW_0 \times \beta^k + \alpha \times t^k)}{(\beta^k + t^k)}$	$\frac{(\alpha - BW_0)k\beta^k t^{(k-1)}}{(\beta^k + t^k)^2}$	$\frac{(\alpha - BW_0)t^{k-1}}{(\beta^k + t^k)}$
6	Richards	$BW_t = \frac{\alpha}{(1 + \beta \times e^{-kt})^{-\frac{1}{m}}}$	$\frac{\alpha k\beta e^{-kt}}{(-m)(1 + \beta e^{-kt})^{(1+\frac{1}{-m})}}$	$\frac{\frac{\alpha}{(1+\beta e^{-kt})^{1/-m}} - \frac{\alpha}{(1+\beta)^{1/-m}}}{t}$

Note: BW_t body weight (g) at time t; BW_0 = initial body weight (g); α = upper asymptotic body weight (g); t = age (weeks); β characterizes the first part of growth before the point of inflection; k describes the second part in which growth rate decreases until the animal reaches the upper asymptotic body weight or mature body weight (α), m is the shape parameter determining the position of the curve point, e – the Euler's number (~ 2.718282).

The predicted body weights (BW) of the ducks were calculated using the predict() function, and then Pearson's correlation (cor) was calculated between the predicted BW and measured BW using the cor() function in the R software.

To determine the optimal slaughter age, the parameters of absolute growth rate (weekly gain, WG_t) and average weekly gain (AWG_t) were determined. Absolute weekly gain ($WG_t = BW_t - BW_{t-1}$) was derived from the first derivative of the mathematical equations presented in **Table 1**. Average weekly gain (AWG_t) was computed as $AWG_t = (BW_t - BW_0)/t$ **(Table 1)**. The maximum AWG_t value (AWG_{max}) was determined as its value when AWGt = WGt (Nguyen Xuan Trach, 2023). The most appropriate slaughter age was defined as the week at which AWG_t reached its maximum (AWG_{max}), aligning with the economic principle of diminishing returns (Drummond & Goodwin, 2004; Nguyen Xuan Trach, 2023). The age at the maximum AWGt estimated based on the best-fit regression function was considered the technically optimal slaughter age (Bo *et al.*, 2023).

Results

Nonlinear growth model selection

All the analyzed models (von Bertalanffy, Janoschek, Gompertz, Logistic, Lopez, and Richards) were able to fit the data satisfactorily (**Table 2**). The coefficients of determination (R^2) for all six non-linear regression functions were greater than 97% for male and 98% for female ducks. This result suggests that the growth of Na Tau ducks could be accurately estimated by any of the six growth functions used in this study. Among all the models, the Janoschek model exhibited the highest R^2 for both male and female ducks.

The Janoschek model had the lowest AIC, BIC, and SER values in both male and female ducks. It is known that the model with the highest R^2 value and the lowest AIC, BIC, and SER is considered the best for describing the growth curve of animals. Therefore, the Janoschek model was the best for describing the growth rate of both male and female ducks. Among the six functions, the Logistic and von Bertalanffy models exhibited lower R^2 values and higher AIC, BIC, and SER values for both males and females. This suggests that these two models were less suitable compared to the other models used in the study for accurately describing the growth curves of Na Tau ducks.

Growth curve parameters

The parameter estimates of the six growth curve models for male and female ducks are presented in **Table 2**. The asymptotic weight parameter (α) values were highest for the Lopez function in both males and females, followed by von Bertalanffy. While the lowest estimates of this parameter were recorded by the Logistic and Janoschek functions. All the models estimated higher values of α for male ducks. The mature asymptotic body weights of male ducks ranged from 2213.16g (Logistic) to 2726.58g (Lopez) and these values in female ducks were estimated from 1968.89g (Logistic) to 2401.25g (Lopez).

The parameters of β and m varied among the models and sexes. The highest values of β were found in the Lopez model for both male and female ducks. The mature growth rate (k) parameter also varied among models, but the values were quite similar between males and females. The k value ranged from 0.03 to 2.27 for males and from 0.04 to 2.22 for females. The highest values of k were obtained in the Lopez model, while the lowest values were found in the Janoschek model for both males and females.

Weight, weight gain estimating, and the suitable slaughter ages of Na Tau ducks

Based on the Janoschek model with parameter estimates from **Table 2**, the body weight (BW_t), weekly gain (WGt), and average weekly gain (AWG_t) of Na Tau males and femals were predicted by age (t) on a weekly basis, as shown below:

For male:

Body weights (BW_t) = $2285.04 - (2285.04-75.38)e^{-0.03t^{1.93}}$

Weekly gain (WG_t) = $(2285.04-75.38) \times 0.03 \times 1.93 \times t^{0.93}e^{-0.03t^{1.93}}$;

Table 2. The growth curve parameters of Na Tau ducks

Beed/Model	α ± SE	β ± SE	k ± SE	m ± SE	BW_0 ± SE	AIC	BIC	SER	R^2
Male									
Bertalanffy	2588.37 ± 35.41	0.85 ± 0.02	0.28 ± 0.01			7406.52 *(5)*	7424.12 *(5)*	37.23 *(4)*	97.61 *(4)*
Janoschek	2285.04 ± 28.83		0.03 ± 0.003	1.93 ± 0.05	75.38 ±15.60	7375.99 *(1)*	7397.99 *(1)*	23.58 *(1)*	97.73 *(1)*
Gompertz	2421.04 ± 25.03	4.01 ± 0.09	0.36 ± 0.01			7380.86 *(2)*	7398.46 *(2)*	24.27 *(2)*	97.71 *(2)*
Logistic	2213.16 ± 15.08		0.62 ± 0.01		119.79 ± 5.20	7409.97 *(6)*	7427.57 *(6)*	42.76 *(6)*	97.60 *(5)*
Lopez	2726.58 ± 64.53	5.48 ± 0.13	2.27 ± 0.08		94.19 ± 15.79	7400.14 *(4)*	7422.14 *(4)*	37.86 *(5)*	97.64 *(3)*
Richards	2420.98 ± 27.13	0.001 ± 0.05	0.36 ± 0.01	-0.0001 ± 0.01		7382.86 *(3)*	7404.86 *(3)*	24.28 *(3)*	97.71 *(2)*
Female									
Bertalanffy	2254.03 ± 23.22	0.85 ± 0.01	0.30 ± 0.01			7614.96 *(5)*	7632.82 *(5)*	32.67 *(5)*	98.29 (5)
Janoschek	2034.74 ± 18.54		0.04 ± 0.002	1.88 ± 0.04	55.80 ± 9.33	7567.65 *(1)*	7589.97 *(1)*	16.37 *(1)*	98.41 *(1)*
Gompertz	2130.64 ± 16.57	3.99 ± 0.08	0.39 ± 0.01			7573.93 *(3)*	7591.79 *(2)*	19.27 *(2)*	98.39 *(3)*
Logistic	1968.89 ± 10.58		0.66 ± 0.01		105.89 ± 4.05	7636.39 *(6)*	7654.25 *(6)*	37.89 *(6)*	98.23 *(6)*
Lopez	2401.25 ± 41.78	5.07 ± 0.09	2.22 ± 0.06		66.95 ± 9.59	7603.96 *(4)*	7626.28 *(4)*	31.03 *(4)*	98.32 *(4)*
Richards	2088.07 ± 23.52	0.84 ± 0.49	0.44 ± 0.02	-0.166 ± 0.08		7571.47 *(2)*	7593.79 *(3)*	19.28 *(3)*	98.40 *(2)*

Note: α = asymptotic body weight (g), which is interpreted as the mature weight; t = age (weeks); β is an integration constant related to the initial animal weight; k is the maturation rate; m is the shape parameter determining the position of the curve point; BW_0: initial body weight (g); AIC: Akaike's information criterion; BIC: Bayesian information criterion; SER: the standard error of the regression function; R^2: Coefficient of determination. Model ranking with AIC, BIC, SER, and R^2 are denoted by an italic number.

Average weekly gain (AWG_t) = $\frac{(2285.04-75.38)(1-e^{-0.03t^{1.93}})}{t}$

For female:

Body weights (BW_t) = 2034.74 – (2034.74-55.80)$e^{-0.04t^{1.88}}$

Weekly gain (WG_t) = (2034.74-55.80) x 0.04 x 1.88 x $t^{0.88}e^{-0.04t^{1.88}}$

Average weekly gain (AWG_t) = $\frac{(2034.74-55.80)(1-e^{-0.04t^{1.88}})}{t}$

The estimated weights by week of age (BW_t), weekly weight gains (WG_t), and average weight gains (AWG_t) based on the Janoschek function for Na Tau males and females are presented in **Table 3**. The weights of Na Tau ducks increased continuously during the study period from one day old to 10 weeks of age. At 10 weeks of age, the weights of the ducks reached 2168.15g for males and 1954.86g for females. Both males and females attained their maximum weekly weight gain at four weeks of age, with males gaining 318.77g and females 301.8g, respectively. The maximum average weight gain, when $WG_t = AWG_t$, were obtained at 6.28 weeks of age when the body weight reached 1617.34g for male; and at 5.81 weeks of age when the body weight reached 1410.9g. As the age at the maximum AWG was considered the technically optimal slaughter age ages for Na Tau ducks. Therefore, the slaughter ages for Na Tau males and females were 6.28 and 5.81 weeks of age, respectively.

Discussion

Growth curves are useful tools for demonstrating how body weight changes during growth and providing relevant and useful data for feeding and breeding programs (Aggrey, 2004). With the ability to predict future growth at any age, growth curves can be used for the pre-selection of animals (Tekel *et al.*, 2005). Growth curves have been described using various mathematical functions. There are numerous studies that have been performed to determine the best growth models in ducks. However, to our knowledge, there are currently only a few on estimating the suitable slaughter age based on descriptions of the growth models. In our study, in addition to determining the best mathematical functions to describe the growth of ducks, we also used the models to predict the suitable slaughter ages for male and female Na Tau ducks.

Nonlinear growth model selection

To know the best-fitting growth model, the coefficients of determination (R^2), Akaike's information criteria (AIC), Schwarz Bayesian information criterion (BIC), and the standard error of the regression function (SER) were evaluated. It is known that the model with the highest R^2 value and the lowest AIC, BIC, and SER is the best model to describe the growth of the animals. In this study, all six estimated growth models could be used to describe the growth curves in Na Tau ducks, with R^2 values higher than 97%. However, to select the best fitting growth models, the parameters of AIC, BIC, and SER were ranked from lowest (= 1) to highest (= 6), and the R^2 ranked from highest (1) to lowest (= 6). The results indicated that the Janoschek function was the best fitting model to describe the growth curves of both male and female ducks. Previous studies on growth function selection for ducks have varied. Vitezica *et al.* (2010) stated that the Weibull model was the best model for mule ducks. Asiamah *et al.* (2020) found that the von Bertalanffy model was the top-performing model to determine the growth patterns of both male and female Leizhou Black ducks, while Thinh *et al.* (2021) indicated that among six growth models, the Gompertz had the best potential for modelling the growth of Eastern spot-billed ducks. The differences in the selection results for the most suitable growth functions among published studies may be due to the duck breeds and animal management practices.

Growth curve parameters

There are various asymptotic weight parameters for different growth models. Parameter α represents the maximum growth response for animals and is primarily influenced by genetic and environmental factors (Narinc *et al.*, 2010; Sariyel *et al.*, 2017). As presented in **Table 2**, the α for both males and females were highest in the Lopez model, while the lowest values were found in the Logistic model. The α of male ducks were higher than those of females in all the models, indicating sexual dimorphism in mature weights. This result also demonstrated that male mature weights were higher than female mature weights. The α of the Na Tau ducks in this study were higher than values found for other duck breeds (Thinh *et al.*, 2021; Padhi *et al.*, 2022). Thinh *et al.* (2021) indicated that Eastern spot-billed ducks in Vietnam had α values from 1179g (Logistic model) to 1397g (Brody model) for males and from 1048g (Logistic model) to 1222g (Brody model) for females. Padhi *et al.* (2022) reported that Kuzi ducks had α values from 1572g (Bridges model) to 1840g (von Bertalanffy). Higher α values suggest that Na Tau ducks in this study reached their adult weights earlier.

An important factor to consider is the growth model parameter k, which is defined as the rate at which a duck grows to reach its mature weight. A duck with a high k value shows advanced maturity (Ghavi Hossein-Zadeh, 2014). Compared to other previous studies on ducks, the k values in this study were quite high. Kokoszyński *et al.* (2019) reported that Poslish Pekin ducks had k values ranging from 0.0001 (Bridges and Janoschek models) to 0.1144 (Logistic model); Thinh *et al.* (2021) found k values in Eastern spot-billed ducks ranging from 0.05 (Janoschek and Bridges models) to 0.6 (Logistic model); and Padhi *et al.* (2022) found the k values of Kuzi ducks ranged from 0.0008 (Bridge model) to 0.0926 (Logistic model).

The suitable slaughter ages of Na Tau ducks

The growth curves of livestock typically reflect a general biological regularity, in which the growth rate begins to slow down at a certain age due to the influence of the law of diminishing returns (LDR). As a result, feed conversion efficiency also declines, since the feed intake must increasingly meet maintenance requirements, while the proportion of nutrients available for growth decreases. This implies that if animals are raised beyond a certain age, their feed conversion ratio (FCR) will increase. Therefore, it is essential to determine the optimal slaughter age at which FCR is minimized in order to reduce feed cost per unit of product (Nguyen Xuan Trach, 2023). The suitable age for slaughter is determined when animals reach their maximum average weekly gain. Several factors including environmental conditions, feed, or genetic factors could contribute to the differences in growth patterns and optimal slaughter ages (Upperman, 2021). Under this study, the most suitable slaughter age of Na Tau males, based on the Janoschek model, was 6.28 weeks when they reached 1617.34g; and the most suitable slaughter age for Na Tau females, based on the Janoschek model, was 5.81 weeks when they reached 1410.9g. The determination of the optimal slaughter age for Na Tau ducks using linear regression functions is of great significance, as it provides a scientific basis for farmers to establish appropriate rearing schedules and avoid prolonged fattening periods that increase production costs without yielding proportional weight gain. Moreover, the regression results reveal sex-related differences in the suitable slaughter age between males and females, thereby offering a reliable foundation for flock management and improving production efficiency.

Conclusions

The best equation for describing the body growth of Na Tau ducks was the Janoscheck model for both males and females. The derived prediction models for males are: body weights (BW_t) = $2285.04 - (2285.04\text{-}75.38)e^{-0.03t^{1.93}}$; weekly gain ($WG_t$) = $(2285.04\text{-}75.38) \times 0.03 \times 1.93 \times t^{0.93}e^{-0.03t^{1.93}}$; average weekly gain ($AWG_t$) = $\frac{(2285.04-75.38)(1-e^{-0.03t^{1.93}})}{t}$. And the derived prediction models for females are: body weights (BW_t) = $2034.74 - (2034.74\text{-}55.80)e^{-0.04t^{1.88}}$; weekly gain ($WG_t$) = $(2034.74\text{-}55.80) \times 0.04 \times 1.88 \times t^{0.88}e^{-0.04t^{1.88}}$; average weekly gain ($AWG_t$) = $\frac{(2034.74-55.80)\,(1-e^{-0.04t^{1.88}})}{t}$. Based on the

Table 3. The actual and predicted body weights (BW, g, Mean ± SD), weekly gains (WG, g), and average weekly gains (AWG, g) of Na Tau ducks using the best model

Week	Males						Females					
	Measured value			Janoschek			Measured value			Janoschek		
	BW	WG	AWG	BW	WG	AWG	BW	WG	AWG	BW	WG	AWG
0	45.33 ± 3.65	-	-	75.38	-	-	44.37 ± 5.37	-	-	55.80		
1	145.43 ± 17.92	100.10	100.10	150.44	75.06	75.06	140.35 ± 15.47	95.98	95.98	137.49	81.69	81.69
2	367.64 ± 48.6	222.21	161.16	347.97	197.53	136.29	358.16 ± 33.95	217.81	156.895	340.52	203.03	142.36
3	642.47 ± 72.44	274.83	199.05	628.18	280.21	184.26	622.89 ± 63.4	264.73	192.84	616.48	275.96	186.89
4	933.52 ± 91.27	291.05	222.05	946.95	318.77	217.89	902.38 ± 83.85	279.49	214.5025	918.28	301.80	215.62
5	1248.78 ± 118.05	315.26	240.69	1263.51	316.56	237.63	1197.09 ± 119.48	294.71	230.544	1206.54	288.26	230.15
6	1531.26 ± 138.33	282.48	247.66	1547.19	283.68	245.30	1446.78 ± 137.41	249.69	233.735	1454.89	248.35	233.18
7	1789.67 ± 127.82	258.41	249.19	1780.45	233.26	243.58	1662.41 ± 121.01	215.63	231.1486	1651.10	196.21	227.90
8	1989.5 ± 130.05	199.83	243.02	1958.13	177.68	235.34	1817.72 ± 96.21	155.31	221.6688	1794.64	143.53	217.35
9	2092.12 ± 140.49	102.62	227.42	2084.30	126.16	223.21	1893.81 ± 81.75	76.09	205.4933	1892.47	97.83	204.07
10	2143.81 ± 147	51.69	209.85	2168.15	83.85	209.28	1937.14 ± 85.41	43.33	189.277	1954.86	62.40	189.91

best-fitting model, the best estimated slaughter ages were 6.28 weeks for males and 5.81 weeks for females. The results obtained from using mathematical functions to describe growth curves in this study could be applied to further feeding and breeding programs of Na Tau ducks.

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