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A New Score Function of IFSs and its Application in the Evaluation of Software Quality

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Abstract

The score function is used to compare intutionistic fuzzy numbers. In this paper, a novel score function of intuitionistic fuzzy sets (IFSs) is investigated. The novel score function was constructed by combining the polynomial and exponential functions working on the degree of membership and degree of non-membership of intuitionistic fuzzy sets. Then, the newly obtained measure overcomes the limitations of some existing score functions. Next, we applied the new measure to construct a method to deal with a multi-criteria decision-making (MCDM) problem. Finally, this MCDM model was used to assess the quality of software projects. The results showed that the new measure was well done and better than other score functions in some cases. This demonstrate the effectiveness of the newly proposed method.

Keywords

Intuitionistic fuzzy set, score function, software projects, MCDM

Introduction

Intuitionistic fuzzy sets (IFSs) were defined by Atanassov (1986). They are a generalization of the fuzzy set (Zadeh, 1965). An intuitionistic fuzzy set (IFS) considers two levels of relevance on an object: one is the membership function and the other is called the non-membership function. Since their inception over 60 years ago, IFSs have become a very effective tool for dealing with many problems in real-world uncertainties such as pattern recognition, clustering, decision-making, classification, etc. Many research results on intuitionistic fuzzy sets have been published, including waste treatment location selection, investigating and choosing desirable cellular mobile telephone service providers, medical diagnosis, waste disposal location selection, medical diagnosis, and solving engineering and agriculture problems (Thao & Duong, 2019; Joshi, 2020; Rani *et al*., 2021; Alkan & Kahraman, 2022).

Along with these, common measurements such as distance and correlation coefficients, and similarity measurements on IFS have

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been identified and widely applied in the problems of decision-making, machine learning, classification, prediction, and pattern recognition. (Garg & Kumar, 2020; Xue & Deng, 2020; Alkan & Kahraman, 2022). Thao & Duong (2019) determined a similarity measure on an intuitive fuzzy set and applied it to the problem of segmenting target markets. Joshi (2020) studied an information measure on IFS and applied it to the problem of detecting errors in a machine. Xue & Deng (2020) applied distance similarity measures on intuitive fuzzy sets to assess feelings of belief. An assessment based on multiple visual blur distances applied to waste disposal site selection was proposed by Alkan & Kahraman (2022). In that same year, Thao & Chou (2022) improved both the entropy and similarity measures of IFSs and applied them to assess the quality of software projects.

Besides the above measures, the comparison of intuitionistic fuzzy numbers has also been studied and applied in many practical problems. Methods for comparing fuzzy numbers are usually implemented through ranking functions or score functions. Xu & Yager (2006) introduced the score function of IFS and applied it in multi-criteria decision-making (MCDM) problems. These score functions have some restrictions (as shown in the next section). As such, the score functions of the intuitionistic fuzzy set have been considered and improved by many researchers and applied in many other fields (Sahin, 2016; Gao *et al*., 2016; Zhang & Xu, 2017; Wang & Chen, 2018; Gong & Ma, 2019). Sahin (2016) proposed a new score function for interval-valued intuitionistic fuzzy sets by taking into account the degree of hesitation of the intuitive fuzzy set. It overcame some of the difficulties that arose in previous methods for determining the rank of interval-valued intuitive fuzzy numbers. However, this score function does not rank well when the intuitive fuzzy set has a degree of membership equal to zero. The same goes for the score function introduced by Zhang & Xu (2017). Gao *et al*. (2016) modified fuzzy entropy and a new scoring function and used them to handle MCDM problems. The major restriction of the score function of Gao *et al*. (2016) is that it cannot be determined when the intuitive fuzzy set has a degree of membership equal to zero. Wang & Chen (2018) defined a new score function and a new precision function of interval-valued intuitive fuzzy values and combined them with a linear programming approach to solve MCDM problems. The limitation of this measure is similar to that of Xu & Yager (2006) in that it is not able to distinguish well the squared difference between membership and non-membership functions that do not change. Gong & Ma (2019) introduced a new score function and accuracy function of interval-valued intuitionistic fuzzy numbers using the fractional function of two amplitudes applied to sorting problem. This score function has overcome the aforementioned disadvantages of the previous score functions but this score function still has the disadvantage of not being able to determine the ranking order of fuzzy numbers whose membership function is equal to the non-member function. However, the score function of Zhang & Xu (2017) performs well in this case.

As mentioned above, many methods of ranking IFSs have been proposed and applied to MCDM but they cannot rank IFSs well because of different limitations. There are core functions that cannot be determined when one of the two components of the intuitionistic fuzzy set is 0. There are also the score functions that cannot determine the rank of two intuitionistic fuzzy sets when their components are equal. These limitations are explored in the section below. There are measures that are still undetermined on some classes of intuitive fuzzy numbers. The reason is that important information affecting the ranking order of intuitive fuzzy numbers has not been considered or the difference between two membership functions of the intuitive fuzzy numbers merely considered. Therefore, if these faulty ranking methods are applied, the decision maker cannot choose the most suitable alternative in MCDM. This is what motivated us to do this research.To solve the above problems, a new score function of the intuitive fuzzy set was proposed.

The objective of this study was to construct a new score function of IFSs. It was built by combining polynomial and exponential functions with variables that are the values of the membership and non-member functions of IFSs. By this method, we obtained a new score function of IFSs which overcame the limitation of some existing score functions. Second, it was used to build a model to solve multi-criteria decision-making problems (MCDM). Finally, this model was used to evaluate software quality projects. Comparison results with other ranking methods have shown that the new method performs effectively with the MCDM problem to evaluate the quality of software.

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Methods

Let X be a universal set.

Definition 1 (Atanassov, 1986): An intuitionistic fuzzy set on X having two components $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$, which are the degrees of membership and the non-membership of the element x in X to A, respectively. They satisfy $\mu_A(x) + \nu_A(x) \le 1$, for all $x \in X$. We denoted an IFS as

$$A = \left\{ \left(x, \mu_A(x), \nu_A(x) \right) \middle| x \in X \right\}.$$

We called $IFS(\mathrm{X})$ a collection of intuitionistic fuzzy sets on X. In which, we have two special IFSs on X, that are $X = \{ (\mathrm{x},1,0) | \mathrm{x} \in \mathrm{X} \}$ and $\varnothing = \{ (\mathrm{x},0,1) | x \in X \}$.

For convenience, in this study, we call $P = (a, b)$ an intuitionistic fuzzy number if $a, b \ge 0$ and $a + b \le 1$.

Definition 2: Let $P_i = (a_i, b_i)$, $(i = 1, 2)$ be two intuitionistic fuzzy numbers:

(1) $P_1 \oplus P_2 = (a_1 + a_2 - a_1 a_2, b_1 b_2)$

(2) $P_1 \otimes P_2 = (a_1 a_2, b_1 + b_2 - b_1 b_2)$

(3) $P_1^\lambda = (a_1^\lambda, 1 - (1 - b_1)^\lambda$ for all the positive real numbers $\lambda > 0$

Given $A, B \in IFS(X)$ are two intuitionistic fuzzy sets we remark that:

Subset: we denote $A \subset B$ and call A a subset of B only if we have $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$.

Equal: we denote $A = B$ and call A equal to B only if we have $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.

Given $X = \{ x_1, x_2, ..., x_n \}$ is a finite universal set.

Definition 3: The entropy of intuitionistic fuzzy sets $A = \left\{ \left(x_i, \mu_A(x_i), \nu_A(x_i) \right) \middle| x_i \in X \right\}$ is determined by

$$E(A) = \frac{1}{2n} \sum_{i=1}^{n} \left[\frac{e^{-|\mu_A(x_i) - \nu_A(x_i)|} - e^{-1}}{1 - e^{-1}} + 1 - \left| \mu_A(x_i) - \nu_A(x_i) \right| \right].$$

Definition 4: We supposed that $\alpha_j = \left(\mu_j, \nu_j \right) (j = 1, 2, ..., n)$ be given intuitionistic fuzzy numbers. The IF weighted geometric (IFWG) operator of them is defined by

$$\mathrm{IF_w}(\alpha_1, ..., \alpha_n) = \left(\prod_{j=1}^{n} \mu_j^{\mathrm{w}_j}, 1 - \prod_{j=1}^{n} (1 - \nu_j)^{\mathrm{w}_j} \right).$$

where $\mathrm{w}_j \in [0,1]$ is the weight of $\alpha_j = \left(\mu_j, \nu_j \right) (j = 1, 2, ..., n)$ and $\sum_{j=1}^{n} \mathrm{w}_j = 1$.

In previous works, to rank the intuitionistic fuzzy numbers, a score function has been used. Now, we recall some score functions to compare intuitionistic fuzzy numbers. Given an intuitionistic fuzzy number $P = (a, b)$, some existing score functions of intuitionistic fuzzy numbers have been published as follows:

Score function of Xu & Yager (2006) $S_X(P) = a - b$ and accuracy function $H_X(P) = a + b$

In 2006, Xu & Yager (2006) gave out the ranking rule of two intuitionistic fuzzy numbers $P_i = (a_i, b_i)$, $(i = 1, 2)$ as follows:

If $S_X(P_1) < S_X(P_2)$ then $P_1 \prec P_2$

If $S_X(P_1) = S_X(P_2)$ then:

+) If $H_X(P_1) < H_X(P_2)$ then $P_1 \prec P_2$

+) If $H_X(P_1) = H_X(P_2)$ then $P_1 = P_2$

The score function of Xu & Yager (2006) has some drawbacks. It can rank more intuitionistic fuzzy numbers in form $P=(a,a+\alpha)$ as $P_1=(0.1,0.4)$ and $P_2=(0,0.3)$. This is because $S_X(P_1)=S_X(P_2)=-0.3$. It needs the help of the accuracy function, in this case $H_X(P_1)=0.5$ > $H_X(P_2)=0.3$, so that $P_1 \succ P_2$.

Over time, several authors have proposed new score functions to improve this situation. We mention the following:

Score function of Sahi (2016) $S_S(P)=a(2-a-b)$

This score function does not work well when the intuitive fuzzy set has a degree of membership equal to zero as shown in **Example 1**. Gao *et al*. (2016) proposed a score function of $S_G(P)$ to avoid this limitation.

Score function of Gao et al. (2016) $S_G(P)=\frac{1}{2}(a-b)(1+\frac{1}{2a})$

It is easy to verify that the score function of Gao *et al*. (2016) was not determined with the intuitionistic fuzzy numbers having the form $(0,b),(0\le b\le 1)$.

Score function of Zhang & Xu (2017)

$$S_Z(P)=\frac{a}{2}(2-a-b)$$

Score function of Wang & Chen (2018) $S_W(P)=a^2-b^2$

Score function of Gong & Ma (2019)

$$S_{GM}(P)=b-a+\frac{a+a^2-b^2}{a+b}$$

Score function of Khan & Ansari (2020)

$$S_K(P)=\frac{1}{2}\left[(a-b)(2-a-b)+1\right]$$

We can verify that with the above score functions, the score functions investigated by Xu & Yager (2006), Gao *et al*. (2016), and Wang & Chen (2018) have values in [-1, 1], while the other score functions obtain positive values in [0, 1]. These sorting functions are popularly applied to evaluating IFSs in MCDM problems. They also have many restrictions. Next, we examine some cases to see some of the limitations of these ranking functions that need to be overcome.

Example 1: We consider two intuitionistic fuzzy numbers $P_1=(0.1,0.4)$ and $P_2=(0,0.3)$. The score function of Sahin (2016) gives $P_1 \succ P_2$ because $S_S(P_1)=0.15>S_S(P_2)=0$, but we get $P_3=(0,0.4)$ and then $S_S(P_2)=S_S(P_3)=0$. This means that the score function of Sahin (2016) does not distinguish between $P_2=(0,0.3)$ and $P_3=(0,0.4)$. The same goes for the measures of Zhang & Xu (2017) and Gong & Ma (2019). The measure of Gao *et al*. (2016) is not able to determine $P_2=(0,0.3)$ and $P_3=(0,0.4)$. The score function of Khan & Ansari (2020) determined $P_1 \succ P_2$ and $P_2 \succ P_3$ (see **Table 1**).

Example 2: We consider two intuitionistic fuzzy numbers $P_4=(0.5,0.5)$ and $P_5=(0.4,0.4)$. The score functions of Xu & Yager (2006), Gao *et al*. (2016), Wang & Chen (2018), Gong & Ma (2019), and Khan & Ansari (2020) do not rank $P_4=(0.5,0.5)$ and $P_5=(0.4,0.4)$. In these cases, the score functions defined by Sahin (2016) and Zhang & Xu (2017) are determined to be $P_4=(0.5,0.5)\succ P_5=(0.4,0.4)$ (see **Table 1**).

Example 3: We consider two intuitionistic fuzzy numbers $P_6=(0.2,0.2)$ and $P_7=(0,0)$. The score functions of Xu & Yager (2006), Sahin (2016), Gao *et al*. (2016), Wang & Chen (2018); Gong & Ma, (2019), and Khan & Ansari (2020) do not rank $P_6=(0.2,0.2)$ and $P_7=(0,0)$. Moreover, the score functions of Gao *et al*. (2016) and Gong & Ma (2019) are not determined when $P_7=(0,0)$. In this case, the score functions of Sahin (2016) and Zhang & Xu (2017) are determined to be $P_6=(0.2,0.2)\succ P_7=(0,0)$ (see **Table 1**).

These limitations have made the application of these measures in MCDM problems sometimes give inaccurate results. It is essential that we work on a new score function that can overcome these disadvantages. Thus, a new rating function is proposed.

Table 1. Comparison of intuitionistic fuzzy numbers based on other ranking functions

Score functions	$P_1=(0.1,0.4)$ $P_2=(0,0.3)$	$P_2=(0,0.3)$ $P_3=(0,0.4)$	$P_4=(0.5,0.5)$ $P_5=(0.4,0.4)$	$P_6=(0.2,0.2)$ $P_7=(0,0)$
S_X	$S_X(P_1)=S_X(P_2)=-0.3$ (*)	$S_X(P_2)=-0.3 > S_X(P_3)=-0.4$	$S_X(P_4)=S_X(P_5)=0$ (*)	$S_X(P_6)=S_X(P_7)=0$ (*)
S_S	$S_S(P_1)=0.15 > S_S(P_2)=0$	$S_S(P_1) = S_S(P_2)=0$ (*)	$S_S(P_4)=0.5 > S_S(P_5)=0.48$	$S_S(P_6)=0.32 > S_S(P_7)=0$
S_G	$S_G(P_1)=-0.9$, $S_G(P_2)=non$ (*)	$S_G(P_2)=non$, $S_G(P_3)=non$ (*)	$S_G(P_4)=S_G(P_5)=0$ (*)	$S_G(P_4)=0$, $S_G(P_5)=non$ (*)
S_Z	$S_Z(P_1)=0.075 > S_Z(P_2)=-0.3$	$S_Z(P_2)=S_Z(P_3)=0$ (*)	$S_Z(P_4)=0.25 > S_Z(P_5)=0.24$	$S_Z(P_6)=0.16 > S_Z(P_7)=0$
S_W	$S_W(P_1)=-0.15 < S_W(P_2)=-0.09$	$S_W(P_2)=-0.09 > S_W(P_3)=-0.16$	$S_W(P_4)=S_W(P_5)=0$ (*)	$S_W(P_6)=S_W(P_7)=0$ (*)
S_{GM}	$S_{GM}(P_1)=0.2 > S_{GM}(P_2)=0$	$S_{GM}(P_2) = S_{GM}(P_3)=0$ (*)	$S_{GM}(P_4)=S_{GM}(P_5)=0.5$ (*)	$S_{GM}(P_6)=0.5$, $S_{GM}(P_7)=non$ (*)
S_K	$S_K(P_1)=0.275 > S_K(P_2)=0.245$	$S_K(P_2)=0.245 > S_K(P_3)=0.18$	$S_K(P_4)=S_K(P_5)=0.5$ (*)	$S_K(P_6)=S_K(P_7)=0.5$ (*)
S_T	$S_T(P_1)=0.1281 > S_T(P_2)=0.124$	$S_T(P_2)=0.124 > S_T(P_2)=0.1036$	$S_T(P_4)=0.2299 > S_T(P_5)=0.2281$	$S_T(P_6)=0.2134 > S_T(P_7)=0.1839$

Note: (*) means that it cannot rank two intuitionistic fuzzy numbers; "non" is not determined.

Results

A new score function of the IFSs

We first noticed that a common property of the score functions listed in the above section:

Property 1: Let $S(a,b)$ be a score function of intuitionistic fuzzy number $P=(a,b)$. For all the intuitionistic fuzzy numbers (a,b), we have:

$S'_a(a,b)\geq 0$

$S'_b(a,b)\leq 0$

We combined both the polynomial and exponential functions in order to construct a new ranking function of intuitive fuzzy numbers. It overcomes the limitations of the previous score functions.

Now, we consider a two real variables function $f:[0,1]^2\to[0,1]$ determined by the formula:

$$f(x,y)=\frac{(1+x-y^2)\exp(x-y-1)}{2} \quad (1)$$

for all $(x,y)\in[0,1]^2$.

It is easy to verify that $f(x,y)$ satisfies property 1 and $0=f(0,1)\leq f(x,y)\leq f(1,0)=1$ for all $(x,y)\in[0,1]^2$.

Definition 5. The core function of intuitionistic fuzzy number $P=(a,b)$ is defined by

$$S_T(a,b)=\frac{(1+a-b^2)\exp(a-b-1)}{2} \quad (2).$$

Definition 6. Given two intuitionistic fuzzy numbers $P_i = (a_i, b_i)$, $(i = 1, 2)$, the ranking rules are determined as follows:

if $S_T(P_1) < S_T(P_2)$ then $P_1 \prec P_2$

if $S_T(P_1) = S_T(P_2)$ then $P_1 = P_2$

This new score function overcomes the limitations of the above score functions (which are mentioned in the preliminary analyses section). The advantage of our proposed score function is shown in **Table 1**. From this table, the disadvantages of the previous measures have been overcome by the proposed new score function. The new measure has a value in the unit interval [0, 1]. Moreover, its minimum value of 0 is only achieved at the valued intuitionistic fuzzy number $A = (0,1)$ and its maximum value of 1 is only obtained at the valued intuitionistic fuzzy number $B = (1,0)$.

Applying the score function to evaluate software quality

In evaluating software product quality, criteria were determined based on ISO standards. We needed to select the best quality software according to these criteria. This was the multi-criteria decision-making (MCDM) problem (Thao & Chou, 2022). The assessment of information is important and necessary. It also depends on the knowledge, capacity of the decision maker, and the information gathered. In this section, we applied the new proposed score function to evaluate software quality. The data were cited in Thao & Chou (2022) (see **Table 2**). The set of attributes was 13 key qualities which were defined by ISO 25010 (ISO, 2017) as C = {Functional Suitability (C_1), Functional Correctness (C_2), Testability (C_3), Performance Efficiency (C_4), Compatibility (C_5), Usability (C_6), Appropriateness Recognizability (C_7), User Interface Aesthetics (C_8), Reliability (C_9), Security (C_{10}), Maintainability (C_{11}), Modifiability (C_{12}), Portability (C_{13})}. There were five software projects SP = $\{SP_1, SP_2, SP_3, SP_4, SP_5\}$. In this data, each software was expressed as an intuitionistic fuzzy set based on the set of criteria C as $SP_i = \left\{ \left(C_j, \mu_{SP_i}(C_j), \nu_{SP_i}(C_j) \right) \middle| C_j \in C \right\}$. We needed to determine the best choice from these five software projects.

To handle this problem, we used the score function described above.

Algorithm 1. Intuitionistic fuzzy set to rank software quality (IFSQ)

Input: The IFS decision matrix of software based on the set of software quality criteria (see **Table 2**).

Output: Ranking of software projects.

The IFSQ has five steps as follows:

Step 1: Compute the entropy of each criterion C_j, $j = 1, 2, ..., 13$ using the entropy of the intuitionistic fuzzy numbers as defined in Definition 3.

$$e_j = E(C_j) \quad (3)$$

where

$$C_j = \left\{ \left(SP_i, \mu_{C_j}(SP_i), \nu_{C_j}(SP_i) \right) \middle| SP_i \in SP \right\}$$

for all $i = 1, 2, ..., 5$ and $j = 1, 2, ..., 13$.

Step 2: Calculate the weight of each criterion C_j, where $j = 1, 2, ..., 13$ by using equation (4)

$$w_j = \frac{1 - e_j}{13 - \sum_{j=1}^{13} e_j} \quad (4)$$

for all $j = 1, 2, ..., 13$.

Step 3: Compute the IF weighted geometric (IFWG) operator of each alternative SP_i $(i = 1, 2, ..., 5)$, which is defined by

$$\text{IF}_w(SP_i) = \left(\prod_{j=1}^{n} \mu_{SP_i}(C_j)^{w_j}, 1 - \prod_{j=1}^{n} (1 - \nu_{SP_i}(C_j))^{w_j} \right) \quad (5)$$

for all $i = 1, 2, ..., 5$, where $w_j \in [0,1]$, $j = 1, 2, ..., 13$ is the weight of C_j as determined in Step 2.

Step 4: Use the score function to determine the score function of each software $S(SP_i) = S(\text{IF}_w(SP_i))$

where $i = 1, 2, ..., 5$.

Step 5. Rank $SP_i \succ SP_k$ if $S(SP_i) > S(SP_k)$ for all $i = 1, 2, ..., 5$.

Now we apply **Algorithm 1** to solve the software quality's problem.

Step 1: Using eq.(3) we get the entropy of each criterion (shown in **Table 3**).

Step 2: Using eq.(4) we can determine the weight of each criterion (see **Table 4**).

Step 3: Compute the IF weighted geometric (IFWG) operator of each alternative SP_i $(i = 1, 2, ..., 5)$ (shown in **Table 5**).

Step 4: Compute the score function of each software $\text{S}(SP_i)\text{=S}(\text{IF}_\text{w}(SP_i))$ where $i = 1, 2, ..., 5$ (see **Figure 1**).

Step 5. Ranking: $SP_3 \succ SP_2 \succ SP_1 \succ SP_5 \succ SP_4$ (see **Figure 2**).

Table 2. The IF decision matrix of the software based on the set of software quality criteria

	C1		C2		C3		C4		C5		C6		C7	
	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν
SP_1	0.49	0.1	0.7	0.16	0.8	0.1	0.81	0.05	1	0	0.25	0.4	0.25	0.4
SP_2	0.6	0.4	0.8	0.01	0.8	0.01	0.9	0.1	0.25	0.1	1	0	0.6	0.04
SP_3	0.36	0.04	0.73	0.03	1	0	1	0	1	0	1	0	1	0
SP_4	0.81	0.05	0.6	0.11	0.49	0.19	0.8	0.01	0.25	0.3	0.25	0.16	0.16	0.18
SP_5	0.25	0.25	0.81	0.05	0.64	0.1	0.81	0.05	0.25	0.3	0.25	0.4	0.81	0.05

Table 2. The IF decision matrix of the software based on the set of software quality criteria (cont.)

	C8		C9		C10		C11		C12		C13	
	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν	μ	ν
SP_1	0.8	0.1	1	0	0.25	0.4	0.49	0.1	0.25	0.4	0.3	0.4
SP_2	0.8	0	0.25	0.1	0.6	0.04	0.6	0.04	1	0	1	0
SP_3	1	0	1	0	1	0	0.36	0.04	1	0	1	0
SP_4	0.49	0.19	0.25	0.3	0.16	0.18	0.81	0.05	0.25	0.16	0.25	0.16
SP_5	0.64	0.1	0.25	0.3	0.81	0.25	0.25	0.25	0.25	0.4	0.25	0.4

Table 3. The entropy of each criterion

	C1	C2	C3	C4	C5	C6	C7
Entropy	0.5505	0.2945	0.2940	0.1471	0.5375	0.5033	0.4738

Table 3. The entropy of each criterion (cont.)

	C8	C9	C10	C11	C12	C13
Entropy	0.2922	0.5375	0.4748	0.5505	0.5033	0.5153

Table 4. The weight of each criterion

	C1	C2	C3	C4	C5	C6	C7
Weight	0.0614	0.0963	0.0964	01164	0.0631	0.0678	0.0718

Table 4. The weight of each criterion (cont.)

	C8	C9	C10	C11	C12	C13
Weight	0.0966	0.0631	0.0717	0.0614	0.0678	0.0662

Table 5. The IF weighted geometric (IFWG) operator of each alternative

	IFWG	
	μ	ν
SP_1	0.5184	0.2074
SP_2	0.6785	0.0376
SP_3	0.8558	0.0079
SP_4	0.3843	0.1154
SP_5	0.4556	0.1991

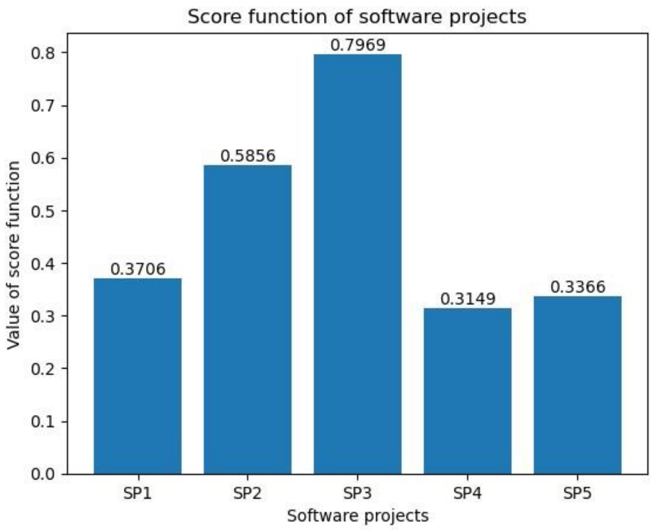

Figure 1. Score of the software projects

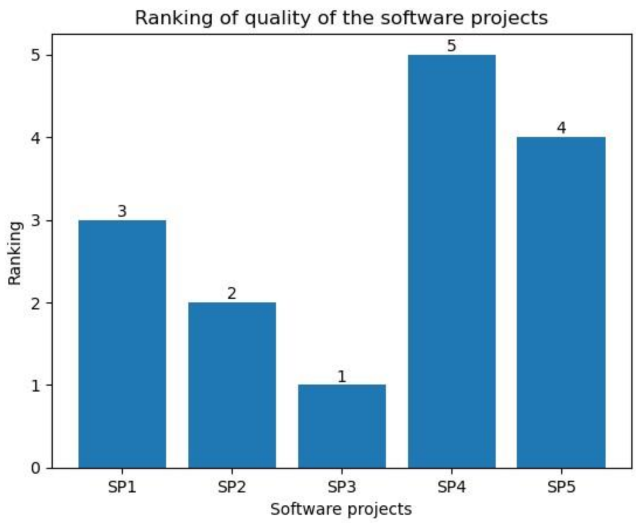

Figure 2. Ranking of the software projects

Discussion

The results from our model indicate that the software project with the highest quality is the third software project SP_3, followed by SP_2, and the last one is SP_4.

In step 4, if we replace our measure by the score functions of Sahin (2016), Gao *et al.* (2016), Zhang & Xu (2017), Wang & Chen (2018), Gong & Ma (2019), or Khan & Ansari (2020) then we obtain the ranking of software projects as shown in **Table 6**. It is easy to see that almost all the score functions give the ranking $SP_3 \succ SP_2 \succ SP_1 \succ SP_5 \succ SP_4$. Only the score function of Gong & Ma (2019) gives the ranking $SP_3 \succ SP_2 \succ SP_1 \succ SP_4 \succ SP_5$.

Furthermore, we also compared this ranking result with some existing ranking methods according to Ye (2016), Zhou (2016), Thao & Duong (2019), Song *et al*. (2019), Thao (2021), and Thao & Chou (2022) to further confirm the feasibility of the new score function and proposed method. The results of this comparison are shown in **Table 7**. According to **Table 7**, we can easily see that all these methods show that SP_3 is the highest, SP_2 is second, and SP_1 is third. There is, however, a difference in the last two ranking orders. Seven of the eight methods indicated that SP_5 and SP_4 ranked fourth and fifth, respectively. Therefore, the final ranking result of the software quality project evaluation problem is $SP_3 \succ SP_2 \succ SP_1 \succ SP_5 \succ SP_4$. The ranking results of the proposed new method also coincide with this ranking.

Conclusions

In this article, we have provided a new score function of intuitionistic fuzzy sets. This measure has overcome the drawback of some existing measures as shown via comparing our proposed measure with previously published measures. Finally, we applied this new score function to build an IFSQ algorithm to evaluate software quality.

In the future, this method could be applied to solve MCDM problems in social economics, agriculture, technology, etc. At the same time, this method could also be used to construct the score function of extension fuzzy sets such as Pythagorean fuzzy sets (Kumar *et al*., 2020; Wang *et al*., 2022), picture fuzzy sets (Dinh *et al*., 2017; Thao & Pham, 2022), and interval-valued

Table 6. Ranking of the software projects using score functions

Software	Proposed	Sahin (2016)	Gao et al. (2016)	Zhang & Xu (2017)	Wang & Chen (2018)	Gong & Ma (2019)	Khan & Ansari (2020)
SP1	3	3	3	3	3	3	3
SP2	2	2	2	2	2	2	2
SP3	1	1	1	1	1	1	1
SP4	5	5	5	5	5	4	5
SP5	4	4	4	4	4	5	4

Table 7. Ranking of the software projects using other methods

Software	Proposed method	Thao & Chou (2022)	Ye (2016)	Thao (2021)	Zhou (2016)	Song *et al.* (2020)	Quynh *et al.* (2020)	Thao & Duong (2021)
SP1	3	3	3	3	3	3	3	3
SP2	2	2	2	2	2	2	2	2
SP3	1	1	1	1	1	1	1	1
SP4	5	5	4	5	5	5	5	5
SP5	4	4	5	4	4	4	4	4

intuitionistic fuzzy sets (Thao & Duong, 2019), which will be promising research projects.

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